# GAS-TO-GAS FILM COOLING*

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Film cooling is a method by which a solid surface is protected from the influence of a hot gas stream in the way that a coolant is locally released from the surface. A cool layer is in this way maintained for some distance downstream of the location of coolant ejection. This cooling method is being used in many advanced engineering applications and it has consequently been studied since 1953 at the Heat-Transfer Laboratory of the University of Minnesota. The following discussion will be concerned with the basic physical processes and analytical approaches as they have evolved in the course of these studies. It will be restricted to situations in which the coolant as well as the fluid in the main stream are gases and where the effect of property variations can be neglected. A complete list of papers and theses published by the Heat-Transfer Laboratory on film cooling is attached. It also contains studies at supersonic velocities and for situations in which the coolant is a different gas than the air in the main stream.

Figure 1 shows various ways in which film cooling is applied. In Fig. 1a the surface, which is to be protected against the hot gas flowing over it, is interrupted by a slot and the coolant gas is ejected through the slot in a downstream direction. Figure 1b shows a similar arrangement, however, with a so-called stepdown slot. In Fig. 1c the coolant gas is ejected through a porous section in the wall and protects in this way the solid portion downstream from the porous section. In Fig. 1d the coolant gas is generated from an ablating material and in 1e from a liquid film so that the generated vapor protects the downstream portion of the wall which is not covered by the liquid film. The ejection in Figs. 1a and b can be through

*This paper is dedicated to the 60th birthday of Academician Aleksei Vasilievich Luikov who, through many years in the past, has contributed significantly to the advancement of international research in heat and mass transfer. The publisher thanks the author for providing the original manuscript.

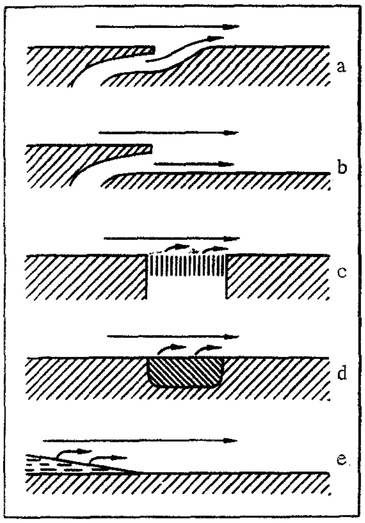


Fig. 1. Various film-cooling systems.

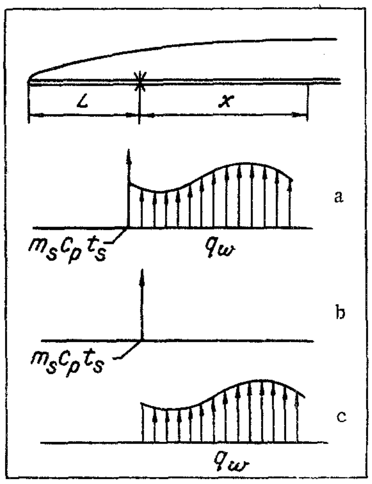


Fig. 2. Heat-source model.

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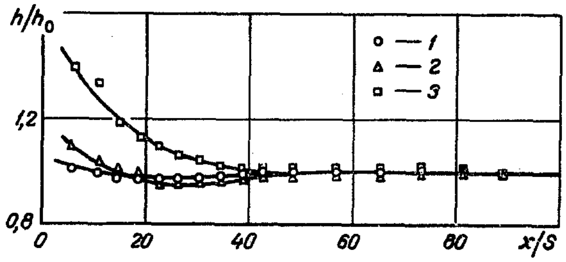


Fig. 3. Ratio of heat-transfer coefficient h with film cooling to heat-transfer coefficient $h_0$ without film cooling as a function of the dimensionless distance x/s. 1) $\rho_s u_s/\rho_e u_e = 0.34$; 2) 0.48; 3) 0.87.

slots which are continuous normal to the flow direction or it can be through a series of interrupted slots or a series of holes. Similar continuous or interrupted ejection can be conceived with the arrangements presented in Figs. 1c and 1d. It has been pointed out that the effectiveness of film cooling decreases in downstream direction and in many applications new slots are provided at a certain distance from the first one in order to renew the coolant layer.

One has, therefore, to deal with a large variety in the geometric configurations of film cooling devices. In addition, the film cooling process will depend on the dimensionless parameters describing the main flow as well as the coolant flow at the point of ejection. In the main flow, the characteristics of the boundary layer at the point of ejection are especially important, described, for instance, by a dimensionless expression for the boundary-layer thickness and by the state of turbulence within the boundary layer. Film cooling is often applied to situations with very large temperature differences and the variation of properties throughout the flow field participating in the heat-transfer process can also become an important factor influencing the effectiveness of film cooling.

The large number of parameters which have, therefore, to be considered require a very extensive program if general quantitative information of the film cooling process is desired. As a consequence, the effect of various of the parameters is today not known accurately. In the present discussion, it will be assumed that the boundary layer at the point of ejection is a fully-developed turbulent one, that the fluids involved are gases and, for most of the discussion, that the temperature differences are sufficiently small so that the thermodynamic and transport properties involved can be considered constant.

## Continuous Ejection

The assumption of constant thermodynamic and transport properties which follows from the conditions listed above introduces already a simplification, namely that the shape of the temperature field and the effectiveness parameter which will be introduced later on are the same whether the ejected fluid is colder or hotter than the fluid of the main stream. This fact has been utilized in experiments which are easier to perform with a heated "coolant." Starting now from the consideration that the temperature field in the boundary layer and especially the temperature of the wall downstream from the slot is the main concern, one can argue that a simplified analysis can be based on a model which simulates the heat addition $m_s c_p t_s$ by the ejected stream into the boundary layer but neglects the influence of mass addition. This means that one replaces the mass ejection through the slot by a line heat source or sink arranged at the slot location [1]. Figure 2 then presents this model with a turbulent boundary layer starting at a distance L upstream of the line heat source whereas x measures the downstream distance from the source. Figure 2a indicates the heat-flux distribution along the surface with a heavy arrow representing the heat release by the line source and the area filled out with small arrows an additional heat release by a heat flux through the wall downstream from the source. As long as properties are considered constant, the energy equation of the turbulent boundary layer is linear. Therefore the law of superposition holds and the situation sketched in Fig. 2a can be analyzed by superposition of the two heat-flux distributions sketched in 2b and 2c. The connection between heat flux and wall surface temperature in Fig. 2c is conventionally described by a heat-transfer coefficient defined through the equation

$$q_w = h(t_w - t_e). \tag{1}$$

The temperature assumed by an adiabatic wall downstream from the slot, as indicated in Fig. 2b, may be denoted by $t_{aw}$ and referred to as adiabatic wall temperature. Superposition of cases 2c and 2b with proper boundary conditions leads after a simple calculation to the result that the wall temperature at the simultaneous presence of the line heat source and the distributed heat sources, as indicated in 2a, is obtained from the equation

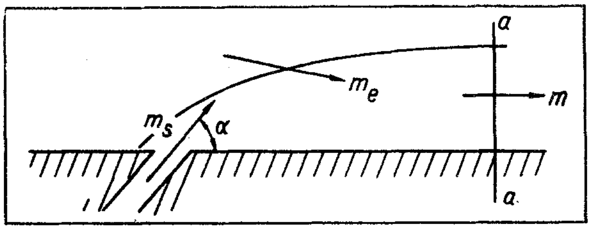


Fig. 4. Symbols for film-cooling analysis.

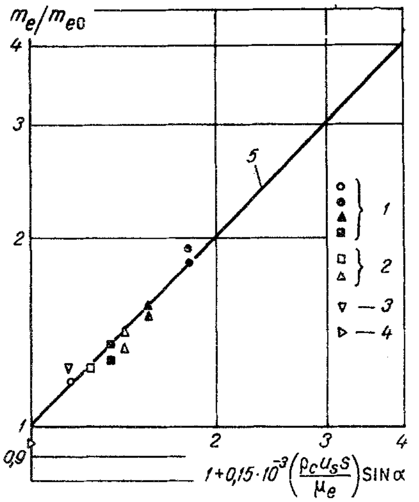


Fig. 5. Experimental results and straight-line correlation for $\beta$ [9]: 1) from [5]; 2) from [28]; 3) from [3]; 4) from [30]; 5) from the formula $m_e/m_{e0} = 1 + 0.15 \cdot 10^{-3} (\rho_c \cdot u_s S/\mu_e) \sin \alpha$.

$$q_w = h\,(t_w - t_{aw}), \tag{2}$$

in which h is the same heat-transfer coefficient as for the case 2c without film cooling, $t_w$ is the actual wall temperature, and $q_w$ the actual heat flux in case 2a and $t_{aw}$ the adiabatic wall temperature for case 2b [1]. This holds for any distribution of $q_w$ along the coordinate x as long as it is the same in Figs. 2a and 2c. The validity of the superposition law for the actual film cooling process has been checked experimentally. Figure 3 presents as an example the ratio of the heat-transfer coefficient h obtained with film cooling (corresponding to case 2a) to the heat-transfer coefficient $h_0$ without film cooling (corresponding to case 2c) plotted over the dimensionless distance x/s downstream from the slot for the film cooling system of Fig. 1a [3]. The parameter on the curves is the ratio of the density $\rho_s$ times exit velocity $u_s$ from the slot to the external mass velocity $\rho_e u_e$ in the main stream outside the boundary layer. The experiments were performed on a flat plate with a constant velocity $u_e$ and a turbulent boundary layer arriving at the slot. It can be observed that, starting with a distance x/s = 22, the two heat-transfer coefficients differ by less than 10% as long as the ratio $\rho_s u_s/\rho_e u_e$ is smaller than 1. An approximamation of this order is often acceptable in engineering calculalations. Additional experiments confirming the superposition rule have been reported in the heat-transfer literature. This means that further investigation can now be restricted to the situation in which the wall downstream from the slot is adiabatic, since information on heat transfer to solid surfaces with the proper heat-flux condition (case 3c) is available.

For the adiabatic condition, some further information can be obtained from the heat source model, applying again the law of superposition. A method [22] is available by which wall temperatures can be calculated for any arbitrary prescribed heat-flux distribution. This method was applied upon my suggestion by Tribus and Klein to the heat source model [22] and the following relation was obtained for a turbulent boundary layer on a flat plate and for a Prandtl number of 0.72

$$\eta_0 = 5.76\,\mathrm{Pr}^{2/3}\,\mathrm{Re}_s^{0,2}\left(\frac{\mu_c}{\mu_e}\right)^{0,2}\frac{c_{pc}}{c_{pe}}\left(\frac{\rho_e u_e x}{m_s}\right)^{-0,8} a, \tag{3}$$

$\eta$ is called film-cooling effectiveness; the slot Reynolds number $\mathrm{Re}_s$ is defined as $m_s/\mu_c$; the parameter $a$ describes the effect of the boundary layer development upstream from the slot. The following relation is obtained by the same method

$$a = \left[1 - \left(\frac{L}{x+L}\right)^{0.9}\right]^{-0,8}. \tag{4}$$

Sufficient experiments, however, are not yet available to judge how well this relation describes the influence of the upstream boundary layer and it will be set equal to one for the following considerations. A differentiation between coolant properties ($\mu_c$, $c_{pc}$) and main-stream properties ($\mu_e$, $c_{pe}$) was achieved through the relation describing the heat flux of the line source in terms of the mass ejection and can be considered as tentative only.

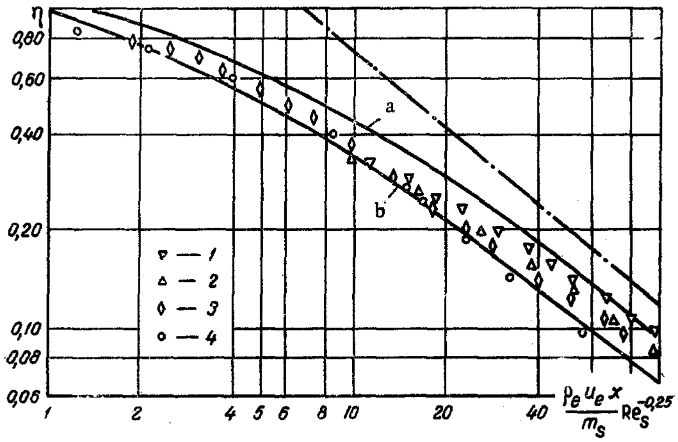


Fig. 6. Film-cooling effectiveness for normal injection [9]. Continuous curves relate to Eq. (16), a for $Re_s = 900$, b for $Re_s = 4500$; dot-dash curve relates to data of Tribus and Klein [22]: 1) Re = 982, $\rho_c u_c/\rho_e u_e = 0.0127$, [19]; 2) 816, 0.0155, [19]; 3) 4444, 0.517, [19]; 4) 4361, 0.40, [17].

Equation (3) when applied to film cooling neglects the effect of mass ejection. One can therefore expect that it agrees best with reality for vanishing values of the dimensionless parameter $m_s/\rho_e u_e x$. This has been verified by the available experiments. A number of analyses have been published which include the effect of mass ejection in order to obtain an expression for the film-cooling effectiveness valid at larger values of this parameter [24, 25, 26]. I will discuss here the analysis by Goldstein and Haji-Sheikh [9] which leads to a good representation of experimental results. The mass flow m within the boundary layer through a plane $a-a$ in Fig. 4 is composed of ejected fluid $m_s$ or fluid $m_e$ entering the boundary layer from the main flow according to

$$m = m_s + m_e. \tag{5}$$

The following energy balance must be fulfilled for steady state and an adiabatic surface when heat conduction through the plane $a-a$ is neglected

$$c_{pc} m_s (t_c - t_e) = c_p m (\bar{t} - t_e). \tag{6}$$

With the expression

$$c_p m = c_{pc} m_s + c_{pe} m_e \tag{7}$$

and a symbol λ as a dimensionless parameter describing the average fluid temperature $\bar{t}$ according to the equation

$$\bar{t} - t_e = \lambda (t_{aw} - t_e) \tag{8}$$

one obtains the following expression for the film-cooling effectiveness

$$\eta = \frac{1/\lambda}{1 + \dfrac{c_{pe} m_e}{c_{pc} m_s}}. \tag{9}$$

The parameter λ is now determined through the requirement that Eq. (3) is an asymptotic expression for this equation at vanishing values of the mass flow $m_s$. For this case Eq. (9) can be simplified to the equation

$$\eta_0 = \frac{c_{pc} m_s}{c_{pe} m_{e0} \lambda}. \tag{10}$$

The subscript zero indicates that this equation holds for vanishing values of the coolant mass flow $m_s$. The symbol $m_{e0}$ denotes then the mass entering the boundary layer from the main stream for zero injection; this means, the mass flow entering a turbulent boundary layer on a solid surface. Expressions for this mass flow can be obtained from the literature, for instance from Eckert and Drake [29],

$$m_{e0} = \frac{7}{8} \rho_e u_e \delta_0, \tag{11}$$

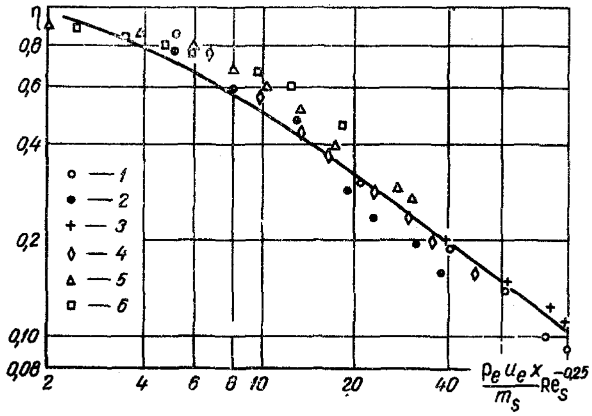


Fig. 7. Film-cooling effectiveness for tangential injection [9]. Curve relates to Eq. (16), the points to randomly selected data from [30, 31]: 1) S = 0.063 inches, $\rho_e u_e/\rho_e u_s$ = 0.18, $\mathrm{Re}_s$ = 620; 2) 0.25, 0.26, 2420; 3) 0.063, 0.39, 1380; 4) 0.125, 0.39, 2720; 5) 0.125, 0.58, 3970; 6) 0.125, 0.88, 6220.

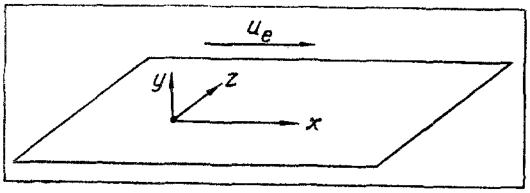


Fig. 8. Coordinate system for point source.

with the boundary-layer thickness $\delta_0$ given by the expression

$$\frac{\delta_0}{x} = \frac{0.376}{(\mathrm{Re}_x)^{1/5}}. \tag{12}$$

Introducing Eqs. (11) and (12) into Eq. (10) results in the following expression for the film-cooling effectiveness

$$\eta_0 = \frac{3.04}{\lambda} \mathrm{Re}_s^{0.2} \left(\frac{\mu_c}{\mu_e}\right)^{0.2} \frac{c_{pc}}{c_{pe}} \left(\frac{\rho_e u_e x}{m_s}\right)^{-0.8} \tag{13}$$

and a comparison of this equation with Eq. (3) (with $a = 1$) indicates the following expression for the parameter $\lambda$

$$\frac{1}{\lambda} = 1.9\,\mathrm{Pr}^{2/3}. \tag{14}$$

It remains to determine the ratio $m_e/m_{e0}$ which will be denoted by $\beta$. In [9] the following expression

$$\beta = \frac{m_e}{m_{e0}} = 1 + 1.5 \cdot 10^{-4}\, \mathrm{Re}_s \frac{\mu_c}{\mu_e} \sin\alpha. \tag{15}$$

has been obtained from the available experimental information. In this equation, $\alpha$ denotes the angle under which the coolant is ejected from the surface as indicated in Fig. 4. Figure 5 shows that this equation represents satisfactorily the available experimental results. The film-cooling effectiveness is then described by:

$$\eta = \frac{1.9\,\mathrm{Pr}^{2/3}}{1 + 0.329\,\mathrm{Re}_s^{-0.2} \frac{c_{pe}}{c_{pc}} \left(\frac{\mu_e}{\mu_c}\right)^{0.2} \left(\frac{\rho_e u_e x}{m_s}\right)^{0.8}}. \tag{16}$$

Figures 6 and 7 compare the film effectiveness obtained by this equation with experimental values for normal ejection of the coolant through a porous strip and for tangential injection through a stepdown slot, respectively. Similar agreement is obtained with experiments by Nishiwaki [27], Wieghardt [28], Hartnett et al. [2, 3], and Eckert and Birkebak [4]. It is, up to now, uncertain whether the influence of variable properties is expressed correctly in Eq. (16) because information from experiments with large temperature differences is not sufficient to verify it.

## Interrupted Ejection

From a stress standpoint, it is often preferable to eject the coolant through a series of holes or of short slots. This ejection method is more difficult to analyze because the velocity and temperature fields involved are 3-dimensional. It is also found in experiments that the ejected coolant stream does not always flow along the surface but penetrates sometimes into the boundary layer or even partially through the boundary layer. The number of configurations with holes or slot arrangements is, of course, very large and this makes it desirable to generalize experimental results by an analytical model. The success of the

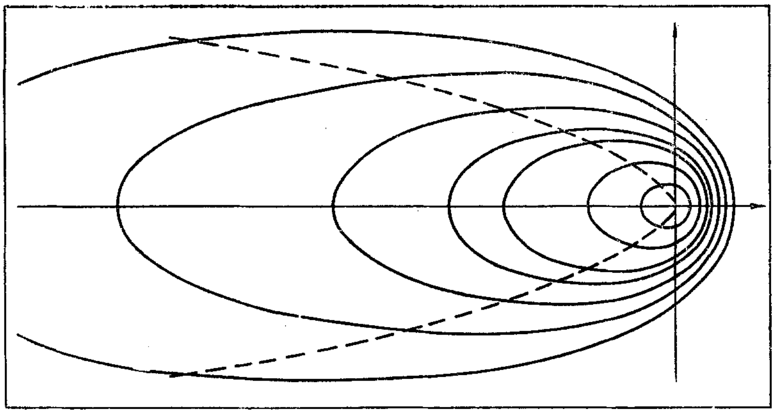

Fig. 9. Temperature field on surface created by point source.

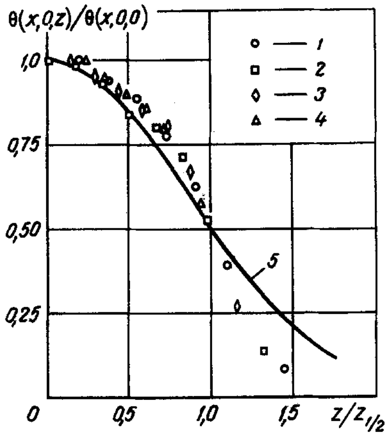


Fig. 10. Variation of adiabatic wall temperature along normals to flow direction for ejection of coolant through a circular hole with axis normal to surface [17], $u_e\rho/\nu_e = 0.9 \cdot 10^5$, $\delta^*/D = 0.05$, $M = 0.1$: 1) $x/D = 1.37$; 2) 3.06; 3) 4.98; 4) 10.07; 5) calculated.

line-source model as representation of continuous ejection suggests to use point heat sources or sinks located properly within the film-cooled surface for the present situation.

A simple analytical expression is available which describes the temperature field created in a semi-infinite solid by a heat source moving along the surface in a straight line with a velocity $u_e$ [29]. The following equation describes the temperature difference $\Theta = t - t_e$, with t denoting the local temperature in the solid.

$$\theta(x, y, z) = \frac{q}{2\pi\rho c_p \alpha r} \exp\left[-\frac{u_e}{2\alpha}(r - x)\right]. \tag{17}$$

The coordinate system used for the point source model is indicated in Fig. 8. The symbol $r = \sqrt{x^2 + y^2 + z^2}$ denotes the distance between the location in the solid with the temperature t and the heat source, the symbol $\alpha$ denotes the thermal diffusivity. The temperature field on the surface of the solid in a coordinate system which moves with the heat source is presented in Fig. 9.

Equation (17) can also be used to describe the temperature field in a fluid moving with uniform velocity and homogeneous turbulence along a surface when a point source is located in the surface. The thermal diffusivity $\alpha$ has in this case to be replaced by the turbulent diffusivity $\varepsilon$. The Eq. (17) can, in addition. be simplified when the dimension x is large compared to y and z. With these changes, the temperature field is described by the equation

$$\frac{\theta(x, y, z)}{\theta(x, 0, 0)} = \exp\left[-\frac{\mathrm{Pe}_t}{4}\left[\left(\frac{y}{x}\right)^2 + \left(\frac{z}{x}\right)^2\right]\right]. \tag{18}$$

The turbulent Peclet number, $\mathrm{Pe}_t$, stands in this equation for the expression $u_e x/\varepsilon$.

The excess temperature on the plate surface along a line in downstream direction starting at the heat source is given by the following equation

$$\theta(x, 0, 0) = \frac{q}{2\pi\rho c_p \varepsilon x}. \tag{19}$$

In applying the point heat-source model to film cooling, the heat flux q issuing from the point source has to be obtained from the relation

$$q = \rho c_p m_h (t_c - t_e). \tag{20}$$

Equation (18) gives an expression for the local film-cooling effectiveness when the coordinate y is set equal to zero. Figure 10 compares this parameter calculated using Eq. (18) with results of measurements which are described in detail in [16]. In the experiments, air was discharged through a cylindrical hole of diameter 2.35 cm with its axis normal to the surface into an air stream which moved with a velocity

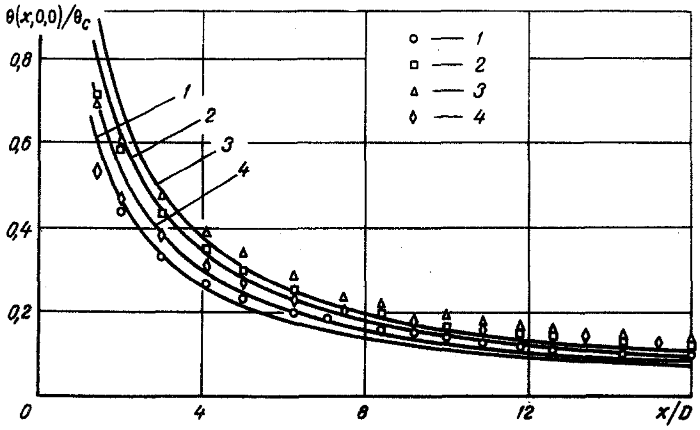


Fig. 11. Variation of adiabatic wall temperature along lines in downstream direction through center of circular hole with axis normal to surface [17], $u_e D/\nu_e = 0.9 \cdot 10^5$, $\delta^*/D = 0.05$: 1) M = 0.1; 2) 0.2; 3) 0.5; 4) 0.75; curves are the calculated values for the respective values of M.

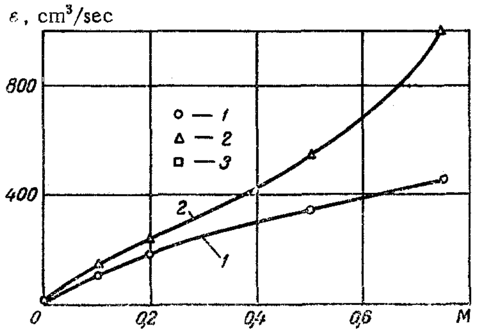


Fig. 12. Thermal diffusivities for jet mixing [17], $u_e$ = 60.7 m/s, D = 2.35 cm, $\delta^*/D$ = 0.05. Single hole with axis normal to surface: 1) diffusivity based on lateral spreading; 2) the same on axial spreading; 3) calculated value (see text).

of 60.7 m/sec over a flat plate creating a turbulent boundary layer which at the location of the injection hole had a displacement thickness equal to 0.05 times the diameter of the injection hole. The Reynolds number based on the hole diameter was $0.9 \cdot 10^5$. Figure 10 presents the temperature, measured at the adiabatic wall on a line normal to the flow direction, normalized by the temperature at the same downstream distance x measured on a line in downstream direction from the center of the injection hole. The coordinate z is normalized by the distance $z_{1/2}$ at which the temperature is 0.5 times the maximum temperature at the same downstream distance x. The data have been obtained at a mass velocity or blowing ratio

$$M = \frac{\rho_c u_c}{\rho_e u_e} = 0.1.$$

It can be observed that in this presentation the temperature profiles measured at various locations x/D correlate on a single curve as required by Eq. (18) and that they agree fairly well with the curve representing Eq. (18). Excess temperatures measured in the adiabatic wall surface along a line directed downstream from the center of the injection hole are plotted in Fig. 11. The excess temperatures have been normalized by the excess temperature $\Theta_c = t_c - t_e$. The measured points can be compared with the curves calculated with Eq. (19). The turbulent diffusivity, ε, required for this calculation has been obtained by a match of the calculated values with the experimental ones at x/D = 3. The analytical values agree again reasonably well with the experimental ones. The fact that at larger values of x/D the experimental values are somewhat higher may be attributed to a probable decrease of the turbulent diffusivity with increasing distance from the injection hole. Figure 12 presents as triangles the diffusivity values obtained from the matching process at x/D = 3. Turbulent diffusivity values may also be obtained from the lateral spreading of the wall surface temperature. The results are inserted in Fig. 12 as the round symbols. It is interesting to note that the diffusivities determined in both ways do not agree for M values of 0.5 and 0.75. This indicates that the heat-source analysis breaks down beyond a certain value of M, probably because the coolant jet has separated from the surface. For the same reason the effectiveness parameter which at first increases with increasing M, decreases again beyond M = 0.5 as can be observed in Fig. 11. Visual observation as well as measured velocity fields in the air stream indicated as well that the cooling air jet lifts off the wall

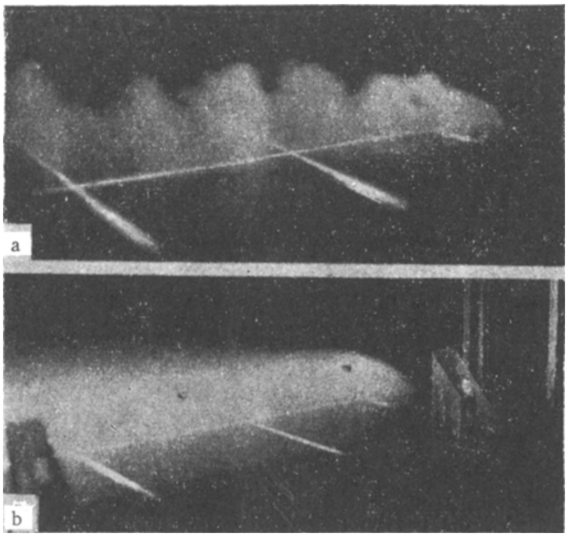


Fig. 13. Flow visualization of jet ejected from circular hole with axis normal to surface mixing with main flow: a) exposure time 1/1000 sec; b) 1/8 sec. M = 9.9.

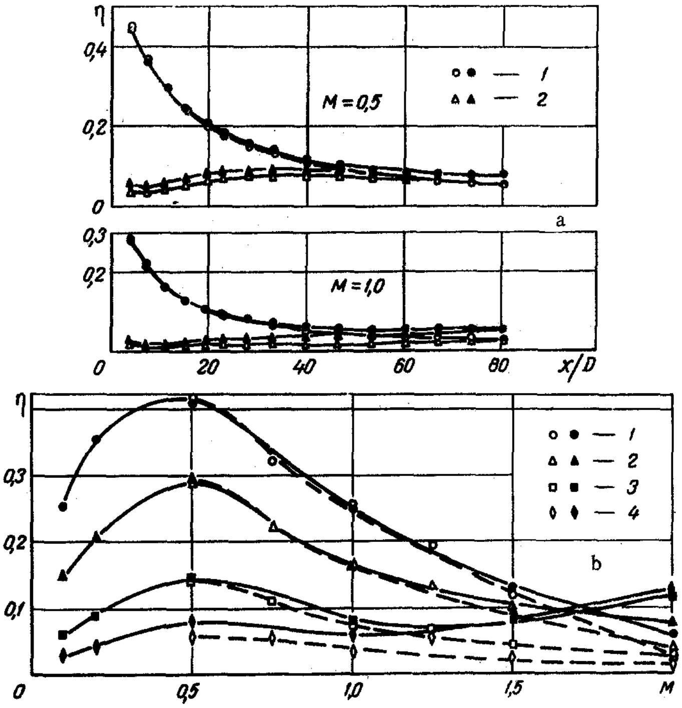


Fig. 14. Film-cooling effectiveness of jets ejected from single hole or transverse row of holes with axis under 35° toward wall surface [21] as related to x/D (a) and M (b) ($u_eD/\nu_e$ = 0.22 · $10^5$; $\delta^*/D$ = 0.124): a) 1) z/D = 0; 2) 1.5; b) z/D = 0: 1) x/D = 5.19; 2) 11.11; 3) 31.47; 4) 80.67. Empty symbols stand for single holes, solid symbols stand for rows of holes of 3D spacing.

surface when a certain value of M is exceeded. It appears from available experimental results that the lift-off occurs around a value M = 0.5.

The turbulent diffusivity values at M = 0.1 and 0.2 at which lift-off has not yet occurred can be compared with turbulent diffusivities reported in the literature for circular or 2-dimensional jets. Values of 150 $cm^2$/sec and of 220 $cm^2$/sec respectively are reported for comparable conditions. The turbulent diffusivity in a normal turbulent boundary layer on a solid flat plate at the corresponding Reynolds number can be calculated to have the value 26 $cm^2$/sec. The values presented in Fig. 12 at M = 0.1 and 0.2 agree quite well with the ones for jets and extrapolation to M = 0 leads to a value which again agrees with the one for the turbulent boundary layer.

An interesting insight into the mixing process of the coolant jet with the main flow has been obtained by flow visualization. Figure 13 presents two photographs of the coolant jet for the lift-off condition: one photo obtained with an exposure time of 1/8 of a second and the other one with 1/1000 of a second. It may be observed that the impression of the jet mixing obtained with a longer exposure time, or for that matter with instruments which average over a longer time period, can be quite misleading. The short-time photo, on the other hand, creates a clear impression of the strong, large-scale turbulence in this mixing process.

The preceding discussion has shown that a point-source model can be used to obtain film-cooling effectiveness values in reasonable agreement with experimental ones for small ejection rates M. Within this range of applicability, the model has considerable advantages because many different film-cooling arrangements can be analyzed simply by superposition of the temperature field as created by a single point source. The temperature fields generated by a transverse row, or even by several rows, of holes as well as by interrupted slots can be obtained in this way. Some experimental results reported in [18] will be used to check on the applicability of this method. The experiments were performed with heated air ejected through a cylindrical hole inclined in downstream direction under an angle of 35° against the surface of a flat plate and through a row of holes inclined under the same angle and arranged in a row normal to the main-flow direction with a spacing of three hole diameters. Film-cooling effectiveness values measured for two locations z/D are plotted in Fig. 14a for two values of the ejection parameter M. The superposition rule requires that the $\eta$ values measured at z/D = 0 are almost equal for the single hole and for the row of holes. This is quite well verified in Fig. 14a. At z/D = 1.5, the effectiveness values should be twice as large for the row of holes as for the single hole, according to the superposition rule. This is in approximate agreement with the experimental values for M = 1. However, it is not the case for M = 0.5. No explanation can be offered for this behavior at the present time and no experiments are presently available for M values smaller than 0.5 for which one would expect the superposition rule to hold. Figure 14b presents the film-cooling effectiveness as a function of the blowing rate M with x/D as parameter. One observes that the film-cooling effectiveness for the single hole as well as for the row of holes increases at first with increasing M and decreases again for M values beyond 0.5. This again indicates the lift-off of the coolant jets occurring at larger blowing rates.

## NOTATION

| | |
|---|---|
| $c_p$ | is the specific heat at constant pressure; |
| D | is the diameter of ejection hole; |
| h | is the film heat-transfer coefficient; |
| L | is the starting length of boundary layer; |
| m | is the mass flow (for the slot per unit length); |
| $M = \rho_c u_c / \rho_e u_e$ | is the blowing rate; |
| $Pe_t = u_e x / \varepsilon$ | is the turbulent Peclet number; |
| Pr | is the Prandtl number; |
| q | is the heat flux; |
| R | is the radial distance; |
| $Re_s = m_s / \mu_s$ | is the slot Reynolds number; |
| $Re_x = \rho_s u_e x / \mu_e$ | is the main-stream Reynolds number; |
| S | is the slot width; |
| t | is the temperature; |
| u | is the velocity; |
| x, y, z | are the coordinates; |
| $\alpha$ | is the ejection angle, thermal diffusivity; |
| $\delta$ | is the boundary-layer thickness; |
| $\delta^*$ | is the displacement thickness; |
| $\varepsilon$ | is the turbulent diffusivity; |
| $\eta = t_{aw} - t_e / t_s - t_e$ | is the film-cooling effectiveness; |
| $\Theta$ | is the excess temperature; |
| $\lambda$ | is the temperature parameter (Eq. 8); |
| $\mu$ | is the viscosity; |
| $\nu$ | is the kinematic viscosity; |
| $\rho$ | is the density. |

## Subscripts

aw denotes the adiabatic wall;
c denotes the coolant;
e denotes the external;
h denotes the hole;
0 denotes at vanishing mass ejection;
s denotes the slot;
w denotes at wall surface.

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